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Representations of $\mathcal{U}_q(SO(5))$ and non-minimal q-deformation

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Abstract. Representations of $SO(5)$ can be constructed on bases such that either the Chevalley triplet (e_1, f_1, h_1) or (e_2, f_2, h_2) has the standard $SU(2)$ matrix elements. The other triplet in each case has a more complicated action. The q-deformation of such representations present striking differences. In one case a *non-minimal* deformation is found to be essential. This is explained and illustrated below. Broader interests of a parallel use of the two bases are pointed out.

The q-deformation of representations of non-simply laced Lie algebras (with roots of unequal length) present special problems. This is illustrated by comparing, for particular cases, the respective q-deformations of irreducible representations of $SO(5)$ in two bases. Imposing the standard $SU(2)$ representations for the triplets of Chevalley generators associated to the shorter and the longer root of $SO(5)$ by turn lead to surprisingly different consequences concerning q-deformation. Irreducible representations of $SO(5)$ are characterized by two invariant parameters n_1 and n_2 ($n_1 \geqslant n_2$, both integer or half-integer). In this paper we will consider only the cases

(i) $n_2 = 0$

and

(ii) $n_2 = n_1$.

Up to now only for these two cases are the solutions complete. But even within such restrictions remarkable features arise. For $n_2 = n_1$ one encounters an example (defined below) of non-minimal q-deformation which is our main result here. The case $n_2 = 0$, needing essentially minimal deformation serves as a contrast. By *minimal* q-deformation we mean [1] introduction of q-brackets for each factor in the classical matrix elements of the Chevalley generators acting on a suitably parametrized set of basis states. Non-minimal means a departure from this involving more subtle and complicated q-deformations of some factors giving back again, of course, the same classical limit. Thus, defining $[x]_p \equiv (q^{px} - q^{-px})/(q^p - q^{-p})$, for any classical factor x the q-deformation

$$x \rightarrow [x]_p$$

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is minimal, while

$$x \to [x_1]_{p_1} - [x_2]_{p_2} \quad \text{with} \quad x = x_1 - x_2$$

is an example of non-minimal deformation. The significances of these definitions will be more explicit after the examples to follow.

The Chevalley generators consist of two triplets (e_1, f_1, h_1), (e_2, f_2, h_2) corresponding to the roots 1 and 2 respectively. The standard Drinfeld–Jimbo construction for $\mathcal{U}_q(SO(5))$ is, with commuting Cartan generators $q^{\pm h_1}$, $q^{\pm h_2}$,

$$\begin{aligned}
&q^{\pm h_1} e_1 = q^{\pm 1} e_1 q^{\pm h_1} \quad q^{\pm h_1} f_1 = q^{\mp 1} f_1 q^{\pm h_1} \\
&q^{\pm 2h_2} e_1 = q^{\mp 1} e_1 q^{\pm 2h_2} \quad q^{\pm 2h_2} f_1 = q^{\pm 1} f_1 q^{\pm 2h_2} \\
&q^{\pm h_1} e_2 = q^{\mp 1} e_2 q^{\pm h_1} \quad q^{\pm h_1} f_2 = q^{\pm 1} f_2 q^{\pm h_1} \\
&q^{\pm h_2} e_2 = q^{\pm 1} e_2 q^{\pm h_2} \quad q^{\pm h_2} f_2 = q^{\mp 1} f_2 q^{\pm h_2} \\
&[e_1, f_2] = 0 \quad [e_2, f_1] = 0 \\
&[e_1, f_1] = [2h_1] \equiv \left(\frac{q^{2h_1} - q^{-2h_1}}{q - q^{-1}} \right) \\
&[e_2, f_2] = [2h_2]_2 \equiv \left(\frac{q^{4h_2} - q^{-4h_2}}{q^2 - q^{-2}} \right) \\
&e_2 e_3^{(\pm)} = q^{\mp 2} e_3^{(\pm)} e_2 \\
&f_3^{(\pm)} f_2 = q^{\mp 2} f_2 f_3^{(\pm)} \\
&[e_1, e_4] = 0 \quad [f_1, f_4] = 0
\end{aligned} \tag{1}$$

where we have defined

$$\begin{aligned}
&e_3^{(\pm)} = q^{\pm 1} e_1 e_2 - q^{\mp 1} e_2 e_1 \\
&f_3^{(\pm)} = q^{\pm 1} f_2 f_1 - q^{\mp 1} f_1 f_2 \\
&e_4 = q^{-1} e_1 e_3^{(+)} - q\, e_3^{(+)} e_1 = q\, e_1 e_3^{(-)} - q^{-1} e_3^{(-)} e_1 \\
&f_4 = q^{-1} f_3^{(+)} f_1 - q\, f_1 f_3^{(+)} = q\, f_3^{(-)} f_1 - q^{-1} f_1 f_3^{(-)}.
\end{aligned} \tag{2}$$

The coproducts, co-units and antipodes are the standard ones.

For subsequent, convenient use we define also

$$\begin{aligned}
&q^{\pm M} = q^{\pm h_1} \quad q^{\pm(K-M)} = q^{\pm 2h_2} \\
&q^{\pm M_2} = q^{\pm\frac{1}{2}(K-M)} = q^{\pm h_2} \\
&q^{\pm M_4} = q^{\pm\frac{1}{2}(K+M)} = q^{\pm(h_1+h_2)}.
\end{aligned} \tag{3}$$

The second-order Casimir operator is [1]

$$\begin{aligned}
A = \frac{1}{[2]}\Big\{ &\big(f_1 e_1 + [M][M+1]\big) \frac{[2K+3]_2}{[2K+3]} + [K][K+3] \Big\} \\
&+ \big(f_2 e_2 + \frac{1}{[2]^2} f_4 e_4\big) + \frac{1}{[2]^2}\big(f_3^{(+)} e_3^{(+)} q^{2M+1} + f_3^{(-)} e_3^{(-)} q^{-2M-1}\big).
\end{aligned} \tag{4}$$

Though we will need in the following only the restricted cases mentioned before we state here the general result that on the space of states spanning the irreducible representation $(n_1,\ n_2)$

$$A = \frac{1}{[2]}\left\{[n_1][n_1+3] + [n_2][n_2+1]\frac{[2n_1+3]_2}{[2n_1+3]}\right\}\cdot\mathbf{1} \tag{5}$$

where $\mathbf{1}$ is the identity. (For $n_2 = 0,\ \frac{1}{2},\ n_1$ this reduces to the results in [1].)

Our aim is to compare two bases for irreducible representations $(n_1,\ n_2)$ defined as follows:

Basis 1. Let

$$\begin{aligned}
&q^{\pm M}|j\ m\ k\ l> = q^{\pm m}|j\ m\ k\ l> \\
&q^{\pm K}|j\ m\ k\ l> = q^{\pm k}|j\ m\ k\ l> \\
&e_1|j\ m\ k\ l> = ([j-m]\ [j+m+1])^{1/2}|j\ m+1\ k\ l> \\
&e_2|j\ m\ k\ l> = \\
&\quad ([j-m+1][j-m+2])^{1/2}\sum_{l'} a(j,k,l,l')|j+1\ m-1\ k+1\ l'> \\
&\quad + ([j+m][j+m-1])^{1/2}\sum_{l'} b(j,k,l,l')|j-1\ m-1\ k+1\ l'> \\
&\quad + ([j+m][j-m+1])^{1/2}\sum_{l'} c(j,k,l,l')|j\ m-1\ k+1\ l'>\,.
\end{aligned} \tag{6}$$

We impose the Hermitian conjugacy $f_i = e_i$ and consider (for generic q) only *real* matrix elements. Hence the matrices realizing e_i and f_i in the space of the representation will be related by transposition.

The domains of the indices have been obtained. The patterns of multiplicities are subtle. They are presented below without the derivations.

(i) For $(n_1,\ n_2)$ integers

$$\begin{aligned}
&j = 0,\ 1, \ldots,\ n_1-1,\ n_1 \\
&m = -j,\ -j+1, \ldots,\ j-1,\ j \\
&k = -l,\ -l+2, \ldots,\ l-2,\ l \\
&l = 0,\ 1,\ 2,\ \ldots \\
&j+l = n_1-n_2,\ n_1-n_2+1, \ldots,\ n_1+n_2 \\
&j-l-\tfrac{1}{2}\left(1-(-1)^{n_1+n_2-j-l}\right) = -n_1+n_2,\ -n_1+n_2+2, \ldots,\ n_1-n_2.
\end{aligned} \tag{7}$$

(When comparing with (2.14) of [1] note that when $n_1 = n_2$, $l = j,\ j-1$ for $j > 0$ and $l = 0$ for $j = 0$.)

(ii) For $(n_1,\ n_2)$ half-integers

$$\begin{aligned}
&j = \tfrac{1}{2},\ \tfrac{3}{2}, \ldots,\ n_1-1,\ n_1 \\
&m = -j,\ -j+1, \ldots,\ j-1,\ j \\
&k = -l,\ -l+1, \ldots,\ l-1,\ l \\
&l = \tfrac{1}{2},\ \tfrac{3}{2}, \ldots \\
&j+l = n_1-n_2+1,\ n_1-n_2+3, \ldots,\ n_1+n_2 \\
&j-l = -n_1+n_2,\ -n_1+n_2+2, \ldots,\ n_1-n_2.
\end{aligned} \tag{8}$$

The domains of k and l were obtained by diagonalizing the J_{45} matrix of Gelfand–Zetlin [2], for low numerical values of n_1 and n_2, by using MATHEMATICA. The general expressions extracted from them were tested again *a posteriori*. At first the domains of k and l came out in a complicated form. Then it was noticed that appropriately combining j and l (as in (7) and (8)) rectangular lattices can obtained as shown.

It can be shown that (7) and (8) lead (for $q = 1$ and generic q) to the same dimensions as the Gelfand–Zetlin construction [2] for $SO(5)$, namely

$$\tfrac{1}{6}(2n_2+1)(2n_1+3)(n_1+n_2+2)(n_1-n_2+1). \tag{9}$$

Up to now the solutions for the reduced matrix elements a, b, c satisfying all the necessary algebraic constraints have been obtained [1] for the cases

$$n_2 = 0,\ \tfrac{1}{2},\ n_1$$

when there is no multiplicity due to l and one can consider states labelled $|j\ m\ k>$. For comparison with the case to follow we reproduce here, briefly, the results for $n_2 = 0$ and $n_1 = n_2$ (for $n_2 = \frac{1}{2}$, see [1]).

To start with, consider only generic q (real, positive). For $n_2 = 0$, $n_1 = n$

$$\begin{aligned}
a(j,k) &= (q+q^{-1})^{-1}\left(\frac{[n-j-k]\,[n+j+k+3]}{[2j+1]\,[2j+3]}\right)^{1/2} \\
b(j,k) &= (q+q^{-1})^{-1}\left(\frac{[n+j-k+1]\,[n-j+k+2]}{[2j-1]\,[2j+1]}\right)^{1/2} \\
c(j,k) &= 0
\end{aligned} \tag{10}$$

where

$$\begin{aligned}
j &= 0, 1, 2, \ldots, n \\
k &= n-j, n-j-2, \ldots, -(n-j-2), -(n-j) \\
m &= j, j-1, \ldots, -(j-1), -j.
\end{aligned}$$

The dimension of the representation is now

$$\tfrac{1}{6}(n+1)(n+2)(2n+3). \tag{11}$$

For $n_2 = n_1 = n$ (integer or half-integer)

$$\begin{aligned}
a(j,k) &= (q+q^{-1})^{-1}\left(\frac{[n-j]_2\,[n+j+2]_2\,[j+k+1]\,[j+k+2]}{[2j+3]\,[2j+1]\,[j+1]_2^2}\right)^{1/2} \\
b(j,k) &= (q+q^{-1})^{-1}\left(\frac{[n-j+1]_2\,[n+j+1]_2\,[j-k]\,[j-k-1]}{[2j+1]\,[2j-1]\,[j]_2^2}\right)^{1/2} \\
c(j,k) &= (q+q^{-1})^{-1}[n+1]_2\frac{([j-k]\,[j+k+1])^{1/2}}{[j+1]_2\,[j]_2}
\end{aligned} \tag{12}$$

where

$$
\begin{aligned}
&j = n, n-1, \cdots, 0\ (1/2)\\
&k = j, j-1, \cdots, -(j-1), -j\\
&m = j, j-1, \cdots, -(j-1), -j.
\end{aligned}
$$

The dimension is

$$\tfrac{1}{3}(n+1)(2n+1)(2n+3). \tag{13}$$

Apart from the limiting values of n_2 mentioned (the lowest 0 and $\frac{1}{2}$ and the highest n_1) not even the *classical* representations have yet been obtained for this basis. (See the detailed discussion and comparison of the situation with that in the Gelfand–Zetlin basis [2] given in [1].) But for the n_2 values mentioned above setting $q = 1$ and comparing with the generic q-case one sees essentially an example of *minimal* q-deformation. The only effect of unequal roots is the appearance of $[x]_2$ brackets along with the $[x]$.

Basis 2. Consider now the following basis states ($\varepsilon = \pm 1$, $\varepsilon' = \pm 1$):

$$
\begin{aligned}
&q^{\pm M_2}|j_2\ m_2\ j_4\ m_4> = q^{\pm m_2}|j_2\ m_2\ j_4\ m_4>\\
&q^{\pm M_4}|j_2\ m_2\ j_4\ m_4> = q^{\pm m_4}|j_2\ m_2\ j_4\ m_4>\\
&e_2|j_2\ m_2\ j_4\ m_4> = ([j_2 - m_2]_2[j_2 + m_2 + 1]_2)^{1/2}|j_2\ m_2 + 1\ j_4\ m_4>\\
&\quad e_1|j_2\ m_2\ j_4\ m_4> = \sum_{\epsilon,\,\epsilon'} ([j_2 - \epsilon\ m_2 + \tfrac{1+\epsilon}{2}]_2)^{1/2} \times\\
&\quad c_{(\epsilon,\epsilon')}(j_2, j_4, m_4)\ |j_2 + \tfrac{\epsilon}{2}\ \ m_2 - \tfrac{1}{2}\ \ j_4 + \tfrac{\epsilon'}{2}\ \ m_4 + \tfrac{1}{2}> .
\end{aligned} \tag{14}
$$

The matrix for f_i is again given by transposing the one for e_i.

The domain of the indices (again for generic q) are

$$
\begin{aligned}
&j_2 = 0,\ \tfrac{1}{2},\ 1, \ldots,\ \frac{n_1 + n_2}{2}\\
&j_4 = 0,\ \tfrac{1}{2},\ 1, \ldots,\ \frac{n_1 + n_2}{2}\\
&m_2 = -j_2,\ -j_2 + 1, \ldots,\ j_2 - 1,\ j_2\\
&m_4 = -j_4,\ -j_4 + 1, \ldots,\ j_4 - 1,\ j_4
\end{aligned}
$$

such that

$$
\begin{aligned}
&j_2 + j_4 = n_2,\ n_2 + 1, \ldots,\ n_1\\
&j_2 - j_4 = -n_2,\ -n_2 + 1, \ldots,\ n_2.
\end{aligned} \tag{15}
$$

Now for $q = 1$ a complete solution for the reduced elements $c_{\epsilon,\epsilon'}$ is available. This is the representation of Hughes [3]. We have introduced the notations j_2, j_4 to indicate the relations of these indices to our definitions in (1) and (2) of (e_2, f_2) and (e_4, f_4) respectively. Moreover the domains of the indices given in [3] is now more simply expressed in terms of the combinations ($j_2 \pm j_4$). (Compare the roles of ($j \pm l$) in (7) and (8).) Though it does not seem to be explicitly noted in the paper, the *shift operators* of [3] correspond directly to the Chevalley generators $(e_1,\ f_1)$. The solutions can be written, in our notations, as

$$c_{(\epsilon,\epsilon')}(j_2, j_4, m_4) = (j_4 + \epsilon' m_4 + \tfrac{1+\epsilon'}{2})^{1/2} c_{(\epsilon,\epsilon')}(j_2, j_4) \quad (\epsilon, \epsilon' = \pm 1) \tag{16}$$

with

$$\begin{aligned} c_{(++)}(j_2,\ j_4) &= c_{(--)}(j_2 + \tfrac{1}{2},\ j_4 + \tfrac{1}{2}) \\ &= \left(\frac{(n_1 + j_2 + j_4 + 3)(n_1 - j_2 - j_4)(j_2 + j_4 + n_2 + 2)(j_2 + j_4 - n_2 + 1)}{(2j_2 + 1)\ (2j_2 + 2)\ (2j_4 + 1)\ (2j_4 + 2)} \right)^{1/2} \end{aligned} \tag{17}$$

and

$$\begin{aligned} c_{(+-)}(j_2,\ j_4) &= -c_{(-+)}(j_2 + \tfrac{1}{2},\ j_4 - \tfrac{1}{2}) \\ &= \left(\frac{(n_1 + j_2 - j_4 + 2)(n_1 - j_2 + j_4 + 1)(j_2 - j_4 + n_2 + 1)(j_4 - j_2 + n_2)}{(2j_2 + 1)\ (2j_2 + 2)\ (2j_4)\ (2j_4 + 1)} \right)^{1/2}. \end{aligned} \tag{18}$$

The dimension is of course, again given by (9).

But now the q-deformation is the problem. As yet solutions have been obtained for the following two cases.

(i) For $n_2 = 0,\ n_1 = n$

$$\begin{aligned} &j_4 = j_2 = 0,\ \tfrac{1}{2},\ \cdots,\ \tfrac{n}{2} \\ &c_{(+-)}(j_2, j_4, m_4) = c_{(-+)}(j_2, j_4, m_4) = 0 \end{aligned}$$

and

$$\begin{aligned} c_{(++)}(j_2,\ j_4,\ m_4) &= ([j_4 + m_4 + 1]_2)^{1/2}\, c_{(++)}(j_2,\ j_4) \\ c_{(--)}(j_2,\ j_4,\ m_4) &= ([j_4 - m_4]_2)^{1/2}\, c_{(--)}(j_2,\ j_4) \end{aligned}$$

where

$$\begin{aligned} c_{(++)}(j_2,\ j_4) &= c_{(--)}(j_2 + \tfrac{1}{2},\ j_4 + \tfrac{1}{2}) \\ &= \left(\frac{[n_1 + 2\, j_2 + 3][n_1 - 2\, j_2]}{[2j_2 + 1]_2[2j_2 + 2]_2} \right)^{1/2}. \end{aligned} \tag{19}$$

This is straightforward. The factorization of m_4-dependence is what one would expect. One has a *minimal* q-deformation (with q^2-brackets appearing as well).

(ii) For $n_1 = n_2 = n$

$$j_2 + j_4 = n,\quad j_2 = 0,\ \tfrac{1}{2}, \ldots,\ n$$

$$c_{(++)}(j_2,\ j_4,\ m_4) = c_{(--)}(j_2,\ j_4,\ m_4) = 0. \tag{20}$$

If one tries to impose for $c_{(\pm,\mp)}$ an m_4-dependence of the type one expects from the classical expression and the typical minimal deformation (found for $n_2 = 0$ say) one runs into a contradiction. The following remarkable solution has been found. One obtains,

$$\begin{aligned} c_{(+-)}(j_2,\ j_4,\ m_4) &= ([n + 1]_2 - [j_2 + m_4 + 1]_2)^{1/2}\, c_{(+-)}(j_2) \\ c_{(-+)}(j_2,\ j_4,\ m_4) &= ([n + 1]_2 - [j_2 - m_4]_2)^{1/2}\, c_{(-+)}(j_2) \end{aligned} \tag{21}$$

where

$$c_{(+-)}(j_2) = -c_{(-+)}(j_2 + \tfrac{1}{2}) = \left(\frac{[2j_2+1]\,[2j_2+2]}{[2j_2+1]_2\,[2j_2+2]_2}\right)^{1/2}. \tag{22}$$

One preserves the correct classical limit. But the m_4-dependence involves a strikingly non-minimal q-deformation prescription. (This, to our knowledge, is the first example of this kind.) One can express the square root of the difference of two brackets (appearing through m_4-dependence) as a square root of products of brackets through the identity

$$[x]_2 - [y]_2 = [x-y]\frac{[x+y]_2}{[x+y]}.$$

But now m_4 appears in the denominator on the right, which is again quite unusual.

From the definition of $(e_4,\ f_4)$ one now obtains (with $j_4 = n - j_2$)

$$\begin{aligned}
&e_4|j_2\, m_2\, j_4\, m_4> = -(q+q^{-1})\,\{([n+1]_2 - [j_2 - m_4]_2)\times \\
&([n+1]_2 - [j_2 + m_4 + 1]_2)\}^{1/2}|j_2\, m_2\, j_4\, m_4 + 1> \\
&\quad f_4|j_2\, m_2\, j_4\, m_4> = -(q+q^{-1})\,\{([n+1]_2 - [j_2 - m_4 + 1]_2)\times \\
&\quad ([n+1]_2 - [j_2 + m_4]_2)\}^{1/2}|j_2\, m_2\, j_4\, m_4 - 1>.
\end{aligned} \tag{23}$$

For comparison we note that for $n_2 = 0$ ($j_2 = j_4$) one has

$$\begin{aligned}
&e_4|j_2\, m_2\, j_4\, m_4> \\
&\quad = (q+q^{-1})\{[j_4 - m_4]_2[j_4 + m_4 + 1]_2\}^{1/2}|j_2\, m_2\, j_4\, m_4 + 1> \\
&f_4|j_2\, m_2\, j_4\, m_4> \\
&\quad = (q+q^{-1})\{[j_4 + m_4]_2[j_4 - m_4 + 1]_2\}^{1/2}|j_2\, m_2\, j_4\, m_4 - 1>.
\end{aligned} \tag{24}$$

Here the classical limit and the $\mathcal{U}_q(SU(2))$ structure associated with $(\frac{e_4}{[2]},\ \frac{f_4}{[2]},\ q^{\pm M_4})$ are evident. For $n_2 = n_1$, the commutator $[e_4,\ f_4]$ is more complicated but, of course, has the same classical limit.

Studying the bases in parallel has other interests than providing interesting exercises in q-deformation. We briefly mention two important aspects to be explored elsewhere.

(a) Suitably adapting familiar continuation techniques $\mathcal{U}_q(SO(3,2))$ and $\mathcal{U}_q(SO(4,1))$ representations can be obtained from basis (1) and basis (2) respectively.

(b) Under suitable contraction procedures again q-deformation of representations of different inhomogeneous algebras are obtained in the two cases. The contractions of basis (1) are discussed in [1]. Contracted representations arising from basis (2) will be presented elsewhere. Here possibilities of applications are particularly interesting.

The major remaining task is the explicit construction of $\mathcal{U}_q(SO(5))$ representations for arbitrary, admissible, $(n_1,\ n_2)$. The elegant formalism of Fiore [4] gives the deformations of only the vector representations of $SO(N)$. If one intends to cover the full range of invariants and indices some essential, hard problems are already encounted at the level of $\mathcal{U}_q(SO(5))$. *Overcoming them is the motivation behind our efforts.*

The basis (1) classical representations seem (so far) to permit relatively simple (minimal) q-*deformation. But the intricate multiplicity patterns (presented here for the first time)* indicate the difficulties of a (even classical) general solution. The unsuitability of the classical Gelfand–Zetlin representations [2] for q-deformation was explained in [1]. The classical representations of Hughes [3] (starting point of our basis (2)) have attractive properties but their q-deformation presents unexpected problems. We hope to present a general solution for basis (2) in a following paper.

The domains of the indices were considered above for generic q. For q a root of unity the situation (concerning dimensions and the centre) changes radically. Nevertheless, the periodic and partially periodic irreducible representations for q a root of unity can be obtained from generic q using our formalism of *fractional parts* [1, 5]†. This will not be discussed here; see section IV of [1] for explanations and references. A different approach, containing supplementary references, is given in [6].

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† The formalism of fractional parts for q a root of unity was first presented in [5].